PROCEEDINGS OF

THE ROYAL SOCIETY.

Section A.—Mathematical and Physical Sciences.

Address of the President, Lord Rayleigh, O.M., D.C.L., at the Anniversary Meeting on November 30, 1908.

Since the last Anniversary the Society has sustained the loss of eighteen Fellows and four Foreign Members.

The deceased Fellows are:—

The Right Hon. Lord Kelvin, died December 17, 1907.
Sir Alfred Baring Garrod, died December 28, 1907.
Robert Lewis John Ellery, died January 14, 1908.
Prof. James Bell Pettigrew, died January 31, 1908.
William Ashwell Shenstone, died February 3, 1908.
Sir John Denis Macdonald, died February 7, 1908.
Lieutenant-General Sir Richard Strachey, died February 12, 1908.
Dr. William Edward Wilson, died March 6, 1908.
Dr. Henry Clifton Sorby, died March 9, 1908.
Sir John Eliot, died March 17, 1908.
The Duke of Devonshire, died March 24, 1908.
Dr. James Bell, died March 31, 1908.
Colonel Andrew Wilson Baird, died April 2, 1908.
Sir John Evans, died May 31, 1908.
Lord Blythswood, died July 8, 1908.
Arthur Lister, died July 20, 1908.
The Earl of Rosse, died August 29, 1908.
Prof. William Edward Ayrton, died November 8, 1908.

The deceased Foreign Members are:—

Pierre Jules César Janssen, died December 23, 1907.
Franz von Leydig, died April, 1908.
Henri Becquerel, died August 25, 1908.
Éleuthère Élie Nicolas Mascart, died August 26, 1908.

The list of deaths this year is exceptionally heavy, and includes the name of one of the most eminent scientific men of our generation, who occupied the Presidency of this Society from 1890 to 1895—I refer, of course, to Lord Kelvin.

We are fortunate in having secured for our 'Proceedings' a review of Kelvin's life and work, written by one who is especially well qualified for the difficult task. I do not doubt that Professor Larmor is right in placing in the forefront of that work those fundamental advances in Thermodynamics which date from the middle of the last century. It was Kelvin who first grasped the full scope of the principle known as the Second Law, a law which may indeed well be considered to stand first in order of importance, regarded from the point of view of man's needs and opportunities. It would be futile to attempt here a re-survey of the ground covered by Professor Larmor.

My acquaintance with Kelvin was limited, until about 1880, a time when I was occupied with measurements relating to the electrical units, and received much appreciated encouragement. From then onwards until his death I enjoyed the privilege of intimacy and, needless to say, profited continually from his conversation, as I had done before from his writings. Our discussions did not always end in agreement, and I remember his admitting that a certain amount of opposition was good for him. Such discussions often invaded the officers' meetings during the time that we were colleagues, not always to the furtherance of the Society's business. But I must not linger over these reminiscences, interesting as they are to me. We shall never see his like.

By the death of Sir Richard Strachey we have lost a man well known to the senior Fellows, who served repeatedly upon the Council and whose advice was always valued. He was a born administrator; and by his work in India and afterwards at the Meteorological Office he rendered splendid service.

Dr. Sorby's researches extended over many fields, and in several of them he was a pioneer. I suppose that his greatest achievement was the introduction of the method in which thin slices of rock are examined under the microscope. Among his many interesting observations are those upon the retardation of freezing in capillary tubes. It appears that the walls exercise

an influence at distances much greater than those usually regarded as molecular—evidence apparently of structure upon an extended scale. Dr. Sorby belonged to a class on whom England has special reason to congratulate herself, men who pursue science unprofessionally. The names of Cavendish, Young, Joule, and Darwin at once suggest themselves. It is to be feared that specialisation and the increasing cost and complication of experimental appliances are having a prejudicial effect in this regard. On the other hand, the amateur is not without advantages which compensate to some extent. Certainly, no one who has the root of the matter in him should be deterred by fears of such difficulties, and the example of Sorby suffices to show how much is open to ingenuity unaided by elaborate appliances.

The name of Sir John Evans must not pass without special notice. There are few in recent years to whom the Society has been more indebted. Many of our Fellows hardly realise how important and laborious are the services rendered in the office of Treasurer. Evans' scientific attainments, his knowledge of the world and of business, and his personal characteristics specially qualified him for office. An appreciation, signed by well-known initials, has recently appeared in our 'Proceedings.'

On the Foreign List also the losses are heavy. We have especially to condole with our colleagues in France upon the havoc caused by death within the last year or two. Janssen, and Mascart, who was much missed at the recent Electrical Conference, had reached a full age. But Becquerel was in the full tide of life, and we had hoped to learn much more from him; as the discoverer of radio-activity, he had opened up inquiries whose significance seems ever on the increase. Science has lost a leader; his friends and the world a charming personality.

During the time that I was Secretary, and so concerned with the passing of mathematical papers through the Press, I was much struck with the carelessness of authors in the arrangement of their manuscript. It is frequently forgotten that a line of print in the 'Transactions' and in the new form of the 'Proceedings' will hold much more than a line of ordinary manuscript, unless, indeed, the handwriting is exceptionally small. Unless the authors' indications were supplemented, it frequently occurred that several lines of print were occupied by what might equally well, and in my judgment much better, be contained in one line. Even practised writers would do well, when they regard their manuscript as complete so far as regards matter and phrasing, to go over it again entirely from the point of view of the printing. In this way much expense and space would be spared, and the appearance of the printed page improved. Professor Larmor has

drawn up a paper which has received the sanction of the Council and is appended to this Address, and will, it is hoped, be of service at once to authors and to the Society.

Apart from questions of printing, the choice of symbols for representing mathematical and physical quantities is of some importance, and is embarrassed by varying usages, especially in different countries. A Committee now sitting is concerned with the selection of symbols for electrical and magnetic quantities, but the question is really much wider. One hesitates to suggest another international conference, and perhaps something could be done by discussion in scientific newspapers. Obviously some give and take would be necessary. When the arguments from convenience are about balanced, appeal might be made to the authority of distinguished men, especially of those who were pioneers in the definition and use of the quantity to be represented. As an example of the difficulties to be faced, I may instance the important case of a symbol for refractive index. In English writings the symbol is usually μ, and on the Continent n. By the early optical writers it would seem that no particular symbol was appropriated. In 1815* Brewster has m. The earliest use of μ that I have come across is by Sir John Herschel,† and the same symbol was used by Coddington (1829) and by Hamilton (1830), both distinguished workers in optics. On the other hand, n was employed by Fraunhofer (1815), and his authority must be reckoned very high. As regards convenience, I should suppose that the balance of advantage would incline to μ, since n is wanted so frequently in other senses. Another case in which there may be difficulties in obtaining a much to be desired uniformity is the symbol for electrical resistance.

On a former occasion I indulged in comment upon the tendency of some recent mathematics, which were doubtless understood as the mild grumbling of an elderly man who does not like to see himself left too far behind. In the same spirit I am inclined to complain of what seem unnecessary changes in mathematical nomenclature. In my youth, by a natural extension of a long established usage relative to equations, we spoke of the *roots* of a function, meaning thereby those values of the argument which cause the *function* to vanish. In many modern writings I read of the *zeroes* of a function in the same sense. There may be reasons for this change; but the new expression seems to need precaution in its use; otherwise we are led to such flowers of speech as "zeroes with real part positive," which I recently came across.‡

* 'Phil. Trans.,' 1815.

† 'Phil. Trans.,' 1821, p. 230.

‡ 'Proc. Math. Soc.,' vol. 31, p. 266.

But though I may use a little my privilege of grumbling over details, I hope I shall not be misunderstood as undervaluing the progress made in recent years, which, indeed, seems to me to be very remarkable and satisfactory, regarded from the scientific point of view. On the other hand I cannot help feeling misgivings as to the suitability of the highly specialised mathematics of the present day for a general intellectual training, and I hope that a careful watch may be maintained to check, in good time, any evil tendencies that may become apparent.

Among the notable advances of the present year is the liquefaction of helium by Professor Onnes of Leiden. It is but a few years since Sir J. Dewar opened up a new field of temperature by his liquefaction of hydrogen, and now a further extension is made which, if reckoned merely in difference of temperature, may appear inconsiderable, but seen from the proper thermodynamical standpoint is recognised to be far-reaching. The exploration of this new field can hardly fail to afford valuable guidance for our ideas concerning the general properties and constitution of matter. Professor Onnes' success is the reward of labours well directed and protracted over many years.

The discovery and application by Rutherford and Geiger of an electrical method of counting the number of α-particles from radio-active substances constitutes an important step, and one that appears to afford better determinations than hitherto of various fundamental quantities. It would be of interest to learn what interpretation is put upon these results by those who still desire to regard matter as homogeneous.

Another very interesting observation published during the year is that of Hale upon the Zeeman effect in sun-spots, tending to show that the spots are fields of intense magnetic force. Anything which promises a clue as to the nature of these mysterious peculiarities of the solar surface is especially welcome. Until we understand better than we do these solar processes, on which our very existence depends, we may do well to cultivate a humbler frame of mind than that indulged in by some of our colleagues.

A theoretical question of importance is raised by the observations of Nordmann and Tikhoff showing a small chromatic displacement of the phase of minimum brightness in the case of certain variable stars. The absence of such an effect has been hitherto the principal argument on the experimental side for assuming a velocity of propagation in vacuum independent of frequency or wave-length. The tendency of the observations would be to suggest a dispersion in the same direction as in ordinary matter, but of almost infinitesimal amount, in view of the immense distances over which the propagation takes place. Lebedew has pointed out that

this conclusion may be evaded by assuming an asymmetry involving colour in the process by which the variability is brought about, and he remarks that although the dispersions indicated by Nordmann and Tikhoff are in the same direction, the amounts calculated from the best available values of the parallaxes differ in the ratio of 30 to 1. In view of this discrepancy and of the extreme minuteness of the dispersion that would be indicated, the probabilities seem at the moment to lie on the side of Lebedew's explanation; doubtless further facts will be available in the near future.

I cannot abstain from including in the achievements of the year the remarkable successes in mechanical flight attained by the brothers Wright, although the interest is rather social and practical than purely scientific. For many years, in fact ever since I became acquainted with the work of Penaud and Wenham, I have leaned to the opinion that flight was possible as a *feat*. This question is now settled, and the tendency may perhaps be to jump too quickly to the conclusion that what can be done as a feat will soon be possible for the purposes of daily life. But there is a very large gap to be bridged over; and the argument urged by Professor Newcomb and based on the principle of dynamical similarity, that the difficulties must increase with the scale of the machines, goes far to preclude the idea that regular ocean service will be conducted by flying machines rather than by ships. But, as the history of science and invention abundantly proves, it is rash to set limits. For special purposes, such as exploration, we may expect to see flying machines in use before many years have passed.

The Report of the National Physical Laboratory for the year again indicates remarkable growth. The various new buildings, which have been erected and equipped during recent years at a cost of about £33,000, are now occupied; and the result is that both researches and test work can be carried out with much greater ease and efficiency than previously. The Executive Committee in charge of the Laboratory is indebted in the first instance to H.M. Government, and then to the numerous friends whose assistance has made this possible. At the same time, the needs for buildings are not nearly satisfied. There has been during the year a very marked and important growth in the demand by manufacturers and others for assistance in metallurgical enquiries, which require investigations, frequently of a very complex character; and with the present accommodation for much of the Metallurgical Department this demand is difficult to satisfy. Thanks in great measure to the Goldsmiths' Company, the chemical side of this department is well provided for; but new buildings for the other branches of metallurgy are an urgent want.

The Report of the Treasury Committee of Inquiry referred to in the address of last year was communicated by the Treasury to the Royal Society, with the intimation that Their Lordships accept the recommendations of the Committee, and trust that the Royal Society may see their way to do the same. In their reply the President and Council, with the concurrence and advice of the Executive Committee of the Laboratory, expressed their readiness to use their best endeavours to carry the Report into effect. The Report has since been presented to Parliament.

The buildings of the Magnetic Observatory at Eskdalemuir are now occupied; but, unfortunately, difficulty has arisen in making the magnetograph rooms which are underground completely watertight, and the recording apparatus is not yet properly installed.

The third and fourth volumes of 'Collected Researches' of the Laboratory have been published during the year, and testify to the vigorous scientific activities of the staff. The third volume is occupied chiefly with the account of the prolonged series of experiments on electric units carried out at the Laboratory by Prof. Ayrton, Mr. Mather, Dr. Lowry, and Mr. Smith. These researches proved of great value in the discussions at the International Conference on Electric Units, for which recently the Society provided accommodation and entertainment at the request of the Government.

The progress of the 'Royal Society Catalogue of Scientific Papers' has advanced a definite stage during the year, through the publication by the Cambridge University Press of the Index Volume of Pure Mathematics for the Nineteenth Century. Owing to the magnitude of the material to be indexed in the several sciences, it has been necessary to adopt drastic measures of compression, and the 40,000 entries involved in the present section have thus been condensed into one royal octavo volume of some 700 pages. An essential element in this saving of bulk has been the grouping of titles within each heading so as to avoid reprinting the leading words. It was, perhaps, inevitable that this device would occasionally be mistaken for an attempt at organic classification within the limits of the main headings, which are substantially those of the yearly 'International Catalogue of Scientific Literature.' This had, indeed, been foreseen in the preface of the volume. As regards new actual sub-headings which have been introduced occasionally, the Committee remark that "These minor classifications, being often made mechanically on the basis of the explicit mention of the sub-heading, are not to be taken as exhaustive; cognate entries may be found elsewhere under the same main heading. The unit of classification is thus the complete numbered heading."

The Committee of the Catalogue have indeed been fully conscious

throughout of the difficulties of the task which they supervise; and it must be gratifying to the Director of the Catalogue and his staff to have the support of high authorities, not confined to this country, in their decision that in so extensive an undertaking practical feasibility must be the aim rather than an elusive theoretical perfection. One advantage, at any rate, will accrue from bringing out a single volume well in advance, in that the Committee will be able to profit in the future work from the experience they have acquired.

Through the kindness of Dr. Schuster I had the opportunity of submitting to the Council, before the expiry of my term of office, a generous proposal which he makes for instituting a fund of £1500, the interest of which is to be applied to pay the travelling expenses of delegates of the Society to the International Association of Academies. Dr. Schuster felt that the absence of such a provision laid a burden upon delegates, and might operate to limit the choice of the Society. I was empowered by the Council to convey their cordial thanks to Dr. Schuster, and I have now the pleasure of making his benefaction known to the Society at large.

In taking leave of the honourable office which I have occupied for three years, I desire to thank the Society and especially my colleagues, the officers, for the consideration which they have uniformly shown me. All the omens indicate that the Society will be represented by one well versed in its affairs, and whose scientific distinction and wide experience justify the highest hopes for his tenure of the chair.

MEDALLISTS, 1908.

Copley Medal.

The Copley Medal is awarded to Dr. Alfred Russel Wallace, F.R.S.

It is now sixty years since this distinguished naturalist began his scientific career. During this long period he has been unceasingly active in the prosecution of natural history studies. As far back as 1848 he accompanied the late Henry Walter Bates to the region of the Amazon, and remained four years there, greatly enriching zoology and botany, and laying at the same time the basis of that wide range of biological acquirement by which all his writings have been characterised. From South America he passed to the Malay Archipelago and spent there some eight fruitful years. It was during his stay in that region that he matured those broad views regarding the geographical distribution of plants and animals which on his return to this country he was able to elaborate in his well-known classic volumes on

that subject. It was there, too, amid the problems presented by the infinite variety of tropical life, that he independently conceived the idea of the theory of the origin of species by natural selection which Charles Darwin had already been working out for years before. His claims to the admiration of all men of science were recognised by the Royal Society forty years ago, when, in 1868, a Royal Medal was awarded to him. Again, when in 1890, the Darwin Medal was founded, he was chosen as its first recipient. He is still full of mental activity and continues to enrich our literature with contributions from his wide store of experience and reflection in the domain of Natural History. As a crowning mark of the high estimation in which the Royal Society holds his services to science, the Copley Medal is now fittingly bestowed on him.

Rumford Medal.

The Rumford Medal is awarded to Prof. H. A. Lorentz, For. Mem. R.S.

Prof. Hendrik Antoon Lorentz, of Leiden, has been distinguished during the last quarter of a century by his fundamental investigations in the principles of the theory of radiation, especially in its electric aspect. His earliest memoirs were concerned with the molecular equivalents which obtain in the refractive (and dispersive) powers of different substances; in them he arrived at formulæ that still remain the accepted mode of theoretical formulation of these phenomena. The main result, that $(\mu^2-1)/(\mu^2+2)$ is proportional jointly to the density of distribution of the molecules, and to a function of the molecular free periods and the period of the radiation in question, rests essentially only on the idea of propagation in some type of elastic medium; and thus it was reached simultaneously, along different special lines, by H. A. Lorentz originally from Helmholtz's form of Maxwell's electric theory, and by L. Lorenz, of Copenhagen, from a general idea of propagation after the manner of elastic solids.

The other advance in physical science with which Prof. Lorentz's name is most closely associated is one of greater precision, the molecular development of Maxwell's theory of electro-dynamics. This subject was never entered upon by Maxwell himself, on the ground, probably, that the general relations of the æther, and in particular their dynamical bearings, offered a definite field which must be fully probed and explored before the uncertainties connected with molecular complexity became ripe for effective detailed treatment. But the theoretical difficulties connected with the simple law of the astronomical aberration of light, and particularly with the entire absence of any effect of the Earth's uniform motion in space on

terrestrial phenomena involving radiation, had more recently rendered this problem urgent. Following on various purely optical papers on the phenomena of moving bodies, Prof. Lorentz, in 1892, elaborated a general molecular treatment in the memoir "La Théorie Electro-magnétique de Maxwell, et son Application aux Corps Mouvants," which appeared in the 'Archives Néerlandaises,' and contains substantially the main root ideas of the subject. In 1905 it was re-expounded with further development in a tract entitled "Versuch einer Theorie der Electrischen und Optischen Erscheinungen in Bewegten Körpern," the main feature being the elimination of the dynamical element in the previous discussion in favour of a formulation by a system of abstract equations, after the way first set out by Maxwell himself as a summary of his final definite results as distinct from the formative ideas underlying them, and afterwards brought into prominence by the expositions of Heaviside and Hertz.

By these writings Prof. Lorentz has taken a predominant place in the modern evolution of electric and optical theory. He has since been active in special applications, of which the best known has been his theoretical prediction of the physical features of the alteration of the lines of the spectrum in a magnetic field, which had been discovered and has since been developed by his colleague Zeeman.

Royal Medals.

The assent of His Majesty the King, our Patron, has been graciously signified to the following awards of the Medals presented annually by him to the Society.

A Royal Medal to Prof. John Milne, F.R.S., for his work on Seismology. In 1875, Dr. Milne accepted the position of Professor at Tokyo, which was offered to him by the Imperial Government of Japan. His attention was almost immediately attracted to the study of earthquakes, and he was led to design new forms of construction for buildings and engineering structures with a view to resisting the destructive effects of shocks. His suggestions have been largely adopted, and his designs have been very successful for the end in view. Incidentally he studied the vibrations of locomotives, and showed how to obtain a more exact balancing of the moving parts, and thus to secure smoother running and a saving of fuel. Here again his suggestions were accepted, and his work was recognised by the Institution of Civil Engineers.

He next devoted himself to the study of artificial shocks produced by the explosion of dynamite in borings. He then studied actual shocks as observed at nine stations connected by telegraph wires. A seismic study of Tokyo, and subsequently of the whole of northern Japan, followed. In this

latter work he relied on reports from 50 stations. The Government then took up the matter, increased his 50 stations to nearly 1000, and founded a Chair of Seismology for Mr. Milne. It is due to his energy, skill, and knowledge that the Japanese School of Seismology stands as the first in the world.

While still in Japan he attempted to obtain international co-operation through the representatives of 13 nationalities. This first effort failed; but subsequently, on his return to England in 1895, he succeeded, and reports are now received by him from some 200 stations furnished with trustworthy instruments, and scattered all over the world. On his return to England he at once established his own observatory at Shide, in the Isle of Wight, and the work has been carried on continuously from that time up to now, mainly by his own industry and resources.

In Great Britain we owe everything in seismology to the British Association. Their Committee was founded in 1880, and since that date Milne has been the moving spirit in the long career of its activity. He has been the author of 29 annual reports, and these form in effect a history of the advance of seismology since it has been recognised as a definite branch of science.

The knowledge which we have now acquired as to the internal constitution of the earth is more due to Milne than to any other man.

The work of Dr. Henry Head, F.R.S., on which is founded the award of the other Royal Medal, forms a connected series of researches on the Nervous System (made partly in conjunction with Campbell, Rivers, Sherren, and Thompson), published for the most part in 'Brain' at various times since 1893 up to the present date, and constituting one of the most original and important contributions to neurological science of recent times.

His first paper (1893), founded on minute and laborious clinical investigation, established in a more precise manner than had hitherto been done the relations between the somatic and visceral systems of nerves. He confirmed from the clinical side the experimental researches of Sherrington on the distribution of the posterior roots of the spinal nerves.

An inquiry into the pathology of Herpes Zoster (1900), which he proved abundantly to be due to inflammation of the posterior root ganglia, indicated that the areas of referred pain in visceral disease corresponded specially with the distribution of the fibres of the posterior roots subserving painful cutaneous sensibility.

Continuing his investigations on the peripheral nerves, partly by experiments on himself, in conjunction with Rivers, and partly by examination of cases of accidental injuries to nerves, Head was led to formulate (1905) an

entirely novel conception and differentiation of the functions of the peripheral nerves, and of the paths for the respective forms of sensibility which they convey—epicritic, protopathic, and deep sensibility. This is generally regarded by neurologists as a research of quite exceptional originality and ability.

Following the course of afferent impulses, Head next showed (1906) that the sensory paths of the peripheral nerves at their first synaptic junction with the spinal cord become re-arranged, and ascend in different relations in certain definite tracts.

Davy Medal.

The Davy Medal is awarded to Prof. William Augustus Tilden, F.R.S.

The researches of Prof. Tilden extend into many domains. His work on the specific heats of the elements in relation to their atomic weights, described to the Society in the Bakerian Lecture for 1900 and in two later papers published also in the 'Philosophical Transactions,' was of high theoretical importance. The employment of liquid oxygen as an ordinary laboratory reagent, rendered possible by the researches of Dewar and others, enabled Prof. Tilden to test the validity of Dulong and Petit's Law and of Neumann's Law over a much wider range of temperature than was possible before, and gave a truer estimate of the nature of their validity.

In the region of organic chemistry, he has carried out important researches on the terpenes, such as that on the hydrocarbons from *Pinus sylvestris*, on terpin and terpinol, and on limettin.

In inorganic chemistry, his investigation on *aqua regia* and on nitrosyl chloride are especially noteworthy. He has assisted much in clearing up many points with regard to *aqua regia* about which obscurity remained. His introduction of nitrogen peroxide and especially of nitrosyl chloride as reagents has proved, in his own hands and in those of other workers, to be of very high value.

Darwin Medal.

The Darwin Medal is awarded to Prof. August Weismann for his contributions to the study of evolution. He was one of the early supporters of the doctrine of evolution by means of natural selection, and wrote in support of the Darwinian theory in 1868. His great series of publications from that date onward must always remain a monument of patient inquiry. In forming an estimate of his work it does not seem essential that we should

decide on the admissibility of his germ-plasm theory. It is in like manner unimportant that he was, in certain respects, forestalled by Galton, and that his own views have undergone changes. The fact remains that he has done more than any other man to focus scientific attention on the mechanism of inheritance. By denying the possibility of somatic inheritance, he has compelled the world to look at this question with a closeness of criticism that is wanting in all earlier inquiries. In the opinion of what is perhaps the majority of naturalists, he has achieved much more than this—he has convinced them that the solution of the problem of evolution must be sought along the lines of his doctrine of germinal continuity. Thus the preformist's point of view, for which he has done so much, forms the basis on which Mendelians and Mutationists are at work.

Weismann's work was highly estimated by Mr. Darwin. Thus he writes, in 1875 ('More Letters,' i, 356), of Weismann's paper on Seasonal Dimorphism: "No one has done so much as you on this important subject, *i.e.*, on the causes of variation." Again ('Life and Letters,' iii, 198): "I have been profoundly interested by your essay on 'Amblystoma,' and think you have removed a great stumbling block in the way of evolution." And, once more, in January, 1877 ('Life and Letters,' iii, 231), Darwin wrote of Weismann's 'Studien zur Descendenzlehre': "They have excited my interest and admiration in the highest degree, and whichever I think of last seems to me the most valuable."

Hughes Medal.

The Hughes Medal is awarded to Prof. Eugen Goldstein.

Prof. Goldstein was one of the early workers on the modern detailed investigation of the electric discharge in rarefied gases, and by long continued researches has contributed substantially to the systematic analysis of the complex actions presenting themselves in that field. Of these researches may be mentioned his observations of the effect of magnetic force on striations, of the phosphorescence produced by the cathode rays, and of the reflection of cathode rays.

By his discovery of the so-called Kanal-Strahlen, or positive rays, he has detected an essential feature of the phenomenon, which, in his own hands and in those of other workers, has already thrown much needed light on the atomic transformations that are involved.

(APPENDIX.)

Practical Suggestions on Mathematical Notation and Printing.

It is a subject of common complaint that mathematical manuscripts are often prepared for press without due regard for the difficulties encountered in setting up the type, or for the appearance of the printed page.

The Council of the Royal Society have had under consideration for some time the desirability of taking steps with a view to diminish the expense of printing and proof-corrections, and to avoid waste of space, and undue variety of notation in papers by different authors in the same volume. They have approved of the reprinting, with modifications and additions, of the substance of a Report to the British Association on this subject, in the hope that greater uniformity and facility in mathematical typography may thereby be promoted. The recommendations which follow are now offered, not in any authoritative way, but simply as a consensus of opinion; to this end it is understood that they were submitted in advance, for consideration and criticism, to the Council of the London Mathematical Society.

*Abstract of Report of British Association Committee.**

With a view to the questions referred to them for consideration, the Committee appointed by the British Association made inquiries into the nature and processes of mathematical printing, and the difficulties attendant thereon; and it appeared to them that a statement of the results of these inquiries would form the best introduction to the suggestions which they had to make.

The process of "composition" of ordinary matter consists in arranging types uniform in height and depth (or "body" as it is termed) in simple straight lines. The complications peculiar to mathematical matter are mainly of two kinds.

First, figures or letters of a smaller size than those to which they are appended have to be set as indices or suffixes; and consequently, except when the expressions are of such frequent occurrence as to make it worth while to have them cast upon type of the various bodies with which they are used, it becomes necessary to fit these smaller types in their proper positions by special methods. This process, which is called "justification," consists in filling up the difference between the bodies of the larger and smaller types with suitable pieces of metal.

* Report of the Committee, consisting of W. Spottiswoode, F.R.S., Prof. Stokes, F.R.S., Prof. Cayley, F.R.S., Prof. Clifford, F.R.S., and J. W. L. Glaisher, F.R.S., appointed to report on Mathematical Notation and Printing, with the view of leading mathematicians to prefer in optional cases such forms as are more easily put into type, and of promoting uniformity of notation.—'B. A. Report,' 1875, pp. 337–339.

The second difficulty arises from the use of lines or "rules" which occur between the numerator and denominator of fractions, and (in one mode of writing) over expressions contained under radical signs. In whatever part of a line such a rule is used, it is necessary to fill up, or compensate, the thickness of it throughout the entire line.

The complications above described may arise in combination or may be repeated more than once in a single expression; and in proportion as the pieces to be "justified" become smaller and more numerous, so do the difficulties of the workman, the time occupied on the work, and the chances of subsequent dislocation of parts augment.

The cost of "composing" mathematical matter may now (1908) in general be estimated at somewhat more than twice that of ordinary or plain matter, the recent adoption of the point system in the casting of types having greatly simplified mathematical justification.

There are many expressions occurring in mathematics which are capable of being written in more than one way; and of these some present much greater difficulties to the printer than others. This being so, the Committee were of opinion that instead of making any specific recommendations, the most useful course they could take would be to append a table of equivalent forms specifying those which do and those which do not involve justification, and also a list of mathematical signs which may fairly be expected to be found, in the usual sizes, ready to hand among a printer's materials.

In recommending in this qualified way some forms of notation in preference to others, the Committee wished it to be distinctly understood that they were speaking from the printing, and not from the scientific point of view; and they were quite aware that, even if some of the easier forms should be adopted in some cases, they may still not be of universal application, and that there may be passages, memoirs, or even whole treatises in which they would be inadmissible.

The Committee drew attention to the advantages which may incidentally accrue to mathematical science by even a partial adoption of the modifications suggested. Anything which tends towards uniformity in notation may be said to tend towards a common language in mathematics; and whatever contributes to cheapening the production of mathematical books must ultimately assist in disseminating a knowledge of the science of which they treat.

MATHEMATICAL SIGNS NOT INVOLVING "JUSTIFICATION."

$$\times \quad - \quad + \quad = \quad \surd \quad \pm \quad :: \quad \therefore \quad \because \quad : \quad \} \quad < \quad > \quad \div$$

$$\Big(\quad \Big[\quad \Big\} \quad \int \quad \Big\surd$$

$$a \quad a' \quad a_1 \quad a^2 \quad a_2 \quad a^{\frac{1}{2}} \quad a_{\frac{1}{2}}$$

EQUIVALENT FORMS

Involving justification.	Not involving justification.
$\dfrac{x}{a}$	x/a or $x \div a$ or $x : a$
$\sqrt{x}$	$\sqrt{}x$ or $x^{\frac{1}{2}}$
$\sqrt[3]{x}$	$\sqrt[3]{}x$ or $x^{\frac{1}{3}}$
$\sqrt{x-y}$	$\sqrt{}(x-y)$ or $(x-y)^{\frac{1}{2}}$
$\sqrt{-1}$	ι or i
$x\,.\,\overline{x+a}$	$x\,(x+a)$
$\epsilon^{\frac{n\pi x}{a}}$	$e^{n\pi x/a}$

This British Association List, which has been abbreviated and modified, is now incorporated in the following:—

RECOMMENDATIONS REGARDING MATHEMATICAL NOTATION AND PRINTING.

Always—

instead of	$\dfrac{x}{3}$	$\dfrac{a+b}{2}$	$\dfrac{a+\dfrac{b}{2}}{\dfrac{c}{3}+\dfrac{d}{4}}$	$\dfrac{a}{b+\dfrac{c}{d}}$	$\sqrt{x}$	$\sqrt{-1}$	$\dfrac{1}{x}$	$\dfrac{1}{x^n}$
write	$\frac{1}{3}x$	$\frac{1}{2}(a+b)$	$\dfrac{a+\frac{1}{2}b}{\frac{1}{3}c+\frac{1}{4}d}$	$\dfrac{a}{b+c/d}$	$\sqrt{}x$ or $x^{\frac{1}{2}}$	ι or i	x^{-1}	x^{-n}

instead of	$x\,.\,\overline{x+a}$	$\sqrt{x-y}$	$e^{\frac{n\pi x}{a}}$	$\int_0^{\frac{\pi}{2}}$	$\lfloor\!\underline{n}$
write	$x\,(x+a)$	$\sqrt{}(x-y)$ or $(x-y)^{\frac{1}{2}}$	$e^{n\pi x/a}$	$\int_0^{\frac{1}{2}\pi}$	$n\,!$

In current ordinary text—

instead of	$\dfrac{x}{a}$	$\dfrac{a+b}{c+d}$	$\dfrac{x}{y+\dfrac{t}{2}}$	$x/y+\dfrac{a}{b+c}$
write	x/a	$(a+b)/(c+d)$	$x/(y+\frac{1}{2}t)$	$\dfrac{x}{y}+\dfrac{a}{b+c}$

Excessive use of the slanting line, or solidus, is, however, undesirable; it may often be avoided by placing several short fractions or formulas, with the intervening words if any, on the same line, instead of setting out each one on a line by itself. The last of the examples given above illustrates an improper use, in which symmetry is spoiled while nothing is gained; either both fractions should be written with the solidus, as $x/y+a/(b+c)$, or else neither as above.

The solidus should be of the same thickness as the horizontal line which it replaces; in some founts of type it is too thick and prominent.

Irregularities in the spacing of letters and symbols in the formulas as printed are often the cause of a general unsatisfactory appearance of the page.

For centimetres, millimetres, kilometres, grammes, kilogrammes, the abbreviations should be cm., mm., km., gm., kgm. (not cms., etc.), and so in similar cases. Present custom is against the use of the signs $\therefore$ and $\because$.

Symbols which are not provided in the usual founts of type are, as a rule, to be avoided. Compounded symbols such as $\dot{a}$ or $\bar{a}$ usually involve justification, and are thus liable to become deranged or broken. The two examples here given have, however, become so essential that separate founts should be provided for them.

The use of a smaller fount for numerical fractions is now customary; thus always $\frac{1}{3}a$ instead of $a/3$. The use of negative exponents often avoids a complex fractional form; as also the use of the fractional exponents, such as $\frac{1}{2}$ and $\frac{1}{3}$. In the latter case $x^{\frac{1}{2}}$ is usually preferred to $x^{1/2}$, notwithstanding that the latter is more legible.

Much is often gained in compactness and clearness by setting out two or more short formulæ on one line, instead of on consecutive lines; in that case they should be separated by spaces, indicated by the sign ♯ on the MS. This would apply with even greater force to expressions such as $x = a, = b, = c$.

In the Preface to his 'Mathematical and Physical Papers,' vol. i, 1880, the late Sir George Stokes successfully introduced the limited use of the solidus notation, obtaining the assent and support of Lord Kelvin, Prof. Clerk Maxwell, Lord Rayleigh, the Editors of the 'Annalen der Physik,' and many other mathematicians. He defined its use as restricted to the symbols immediately on the two sides of it, unless a brace or stop intervenes; thus $\sin n\pi x/a$ is to mean $\sin(n\pi x/a)$; but $\sin n\theta./r^n$, in case it is used, would mean $(\sin n\theta)/r^n$.